**On The Causes And Cures Of Audio Distortion**
**Of Received AM Signals Due To Fading**

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The purpose of this article is to investigate some of the causes and cures of audio distortion of received AM signals due to fading. In the literature on distortion in AM diode detectors it is commonly said that the cause of distortion of received AM signals, without fading, is diode non-linearities. We begin with such a development.

An AM radio signal consists of a carrier signal of frequency $f_c$ together with two sidebands containing numerous signals of varying frequencies. The sideband of frequencies less than the carrier frequency is a mirror image of the sideband above the carrier frequency. To simplify the discussion, and as a first approximation, let us suppose the audio part of the AM signal consists of a single tone of constant (unchanging) amplitude A and constant (unchanging) frequency f. In this case, the amplitude as a function of time, denoted y(t), is given by

$$y(t) = [1 + m\cos(\omega t)]A\cos(\omega_c t) .$$

where $\omega_c = 2\pi f_c$, $\omega = 2\pi f$, and m is the modulation index, where $0 \le m \le 1$. For a receiver with a bandwidth greater than 2f, AM diode detection with the carrier at the center of the filter bandpass is usually approximated by the square law for diode detectors:

$$[y(t)]^2 = [1 + m\cos(\omega t)]^2A^2\cos^2(\omega_c t) .$$

However, this approximation of diode detection is not correct because the square law approximates full wave rectification, while diode detection is inherently half wave rectification. For a correct approximation of diode detection of an AM signal, the square of the signal (above) should be multiplied by the Fourier series for a square wave switched by the carrier:

$$sw(t) = 2/\pi\ [\pi/4 + \cos(\omega_c t) - 1/3\cos(3\omega_c t) + 1/5\cos(5\omega_c t) - \ ...] .$$

In previous versions a half wave Fourier series was used, but the half wave model did not give a correct account of fading distortion when the AM carrier is off tuned. Thus we have the following:

$$[y(t)]^2\, hw(t) = \{[1 + m\cos(\omega t)]^2A^2\cos^2(\omega_c t)\}$$

$$\{2/\pi\ [\pi/4 + \cos(\omega_c t) - 1/3\cos(3\omega_c t) + 1/5\cos(5\omega_c t) - \ ...]\} .$$

Throughout these notes we will use the the following trig identities as needed.

$$\cos^2(x) = 1/2 + 1/2\cos(2x), \text{ and}$$

$$\cos(x)\cos(y) = 1/2\cos(x - y) + 1/2\cos(x + y) .$$

Using the first identity above, we get

$$[y(t)]^2\, sw(t) = \{[1 + m\cos(\omega t)]^2A^2[1/2 + 1/2\cos(2\omega_c t)]\}$$

$$\{2/\pi\ [\pi/4 + \cos(\omega_c t) - 1/3\cos(3\omega_c t) + 1/5\cos(5\omega_c t) - \ ...]\} .$$

When these products are expanded and simplified using the trig identities above we get the following:

$$[y(t)]^2\, sw(t) = mA^2/2\cos(\omega t) + m^2A^2/8\cos(2\omega t) + \text{constant terms}$$

$$+ \text{terms containing } \cos(m(n\omega_c \text{ +/- } \omega)t) .$$

Audio frequencies f are much lower (less) than carrier frequencies $f_c$, and AM (diode) detectors are followed by a low pass filter to attenuate the RF (higher) frequencies and a coupling capacitor to remove the constant terms, so that all that remains is

$$\text{audio}(t) = mA^2/2\cos(\omega t) + m^2A^2/8\cos(2\omega t)$$

which is the desired AM audio frequency f plus harmonic distortion of frequency 2f.

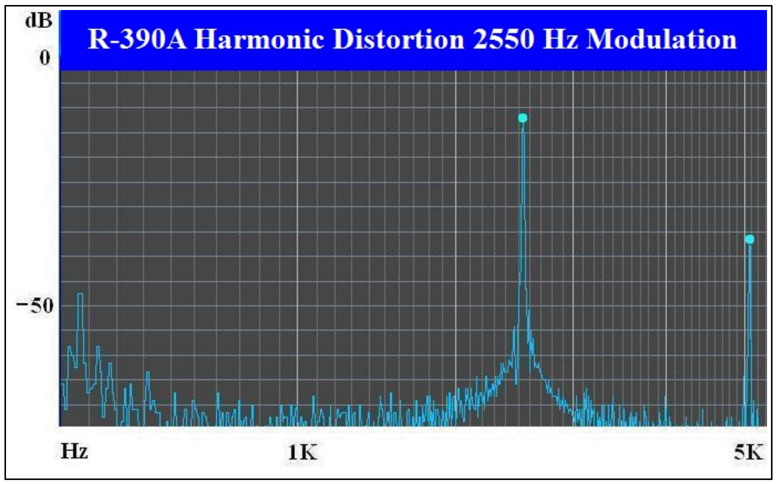
The percent harmonic distortion is given by

$$[(m^2A^2/8)/(mA^2/2)]^2 * 100 = ( m^2/16) * 100 = 6.25\ m^2 .$$

For 50% modulation the percent harmonic distortion due to the diode detector is 1.5625%, which is 18 dB below the original audio signal.

To study the validity of the theoretical development above the harmonic distortion of a steady state (not fading) AM signal was measured for a modified R-390A with a 6.5 kHz bandwidth filter. The modulated input to the R-390A was generated with an HP 8640B set for about 1400 kHz and modulated at 50% with an HP 651B set for about 2550 Hz. The R-390A was tuned to about 1400 kHz, i.e., with the carrier tuned to the center of the R-390A 6.5 kHz filter passband. The R-390A audio output was recorded with a Sony MZ-N510 MD recorder and converted to a .WAV file using WavePad. The graph in the figure below was made from a stop action file of a FFT spectrum analysis of the R-390A audio output.

As is seen in the graph, the 5100 Hz harmonic distortion due to steady state simulation is about 25 dB below the fundamental, which is about 7 dB lower than was predicted above by the square law model. This demonstrates that the square law model is not necessarily an accurate predictor of steady state (not fading) harmonic distortion.




So much for theory and practice.

Nevertheless, it is interesting to continue the theoretical development if only because it may suggest potential ways for reducing AM distortion, both without and with fading.

A theoretical development similar to the one above was made for the case of off tuning, namely tuning an AM signal so that its carrier is near one skirt or the other of the filter passband, mostly attenuating one sideband, from which the following audio output approximation was derived:

$$[y(t)]^2 = \{[1 + m \cos(\omega t)]A\cos(\omega_c t)\}^2$$

$$= \{A\cos(\omega_c t) + mA \cos(\omega t)\cos(\omega_c t)\}^2$$
$$= \{1/2\ mA \cos((\omega - \omega_c)t) + A\cos(\omega_c t) + 1/2\ mA \cos((\omega + \omega_c)t)\}^2$$

and if the upper sideband is mostly attenuated, we may approximate that case by

$$[y(t)]^2 = \{1/2\ mA \cos((\omega - \omega_c)t) + A\cos(\omega_c t)\}^2$$

followed by multiplication by the square wave Fourier series

$$[y(t)]^2\ sw(t) = \{1/2\ mA \cos((\omega - \omega_c)t) + A\cos(\omega_c t)\}^2$$

$$\{2/\pi\ [\pi/4 + \cos(\omega_c t) - 1/3 \cos(3\omega_c t) + 1/5 \cos(5\omega_c t) - \ ...]\}$$

which after the usual expansion, application of trig identities, and collection ot terms the following is obtained.

$$audio(t) = mA^2/4 \cos(\omega t) .$$

So according to the theory above there is no steady state (not fading) harmonic distortion when maximum off tuning is used. But in practice, this was not the case. Off tuning an R-390A using the test setup described above did not result in any reduction of the steady state harmonic distortion. Instead, at some offsets the steady state harmonic distortion increased by up to 6 dB. The tube audio amplifiers of an R-390A surely contribute substantial harmonic distortion to the audio output, so perhaps there was some distortion cancellation in the previous case, and in this case the harmonic distortion might be entirely due to the audio amplifiers.

To develop a mathematical model of fading distortion with the AM carrier centered in the receiver filter bandpass the initial mathematical starting point was modified as follows:

$$[y(t)]^2\, sw(t) = \{mA/2 \cos((\omega_c - \omega)t) + kA\cos(\omega_c t) + mA/2 \cos((\omega_c + \omega)t)\}^2$$

$$\{2/\pi\, [\pi/4 + \cos(\omega_c t) - 1/3 \cos(3\omega_c t) + 1/5 \cos(5\omega_c t) - \ldots]\}$$

where k is a constant less than 1 (which determines how much the carrier has faded relative to the sidebands). After multiplying by the half wave Fourier series, low pass filtering, and so on all that remains is:

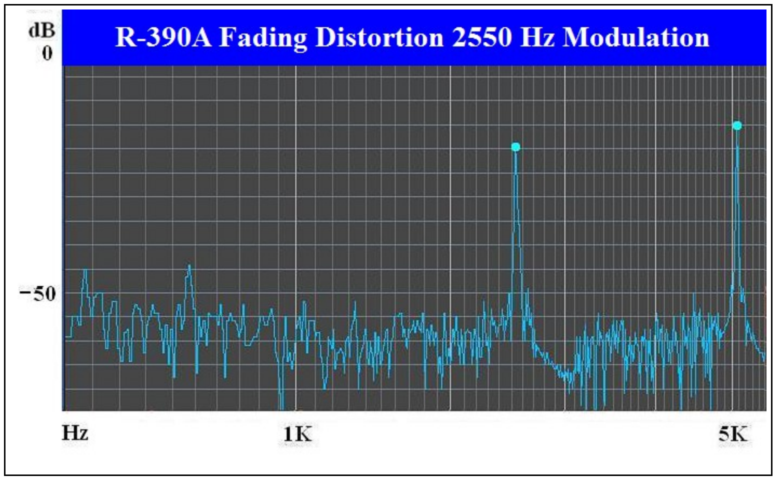
$$\text{audio}(t) = kmA^2/2 \cos(\omega t) + m^2A^2/8 \cos(2\omega t)\ .$$

From this it follows that the percent harmonic distortion due to fading of the carrier is given by:

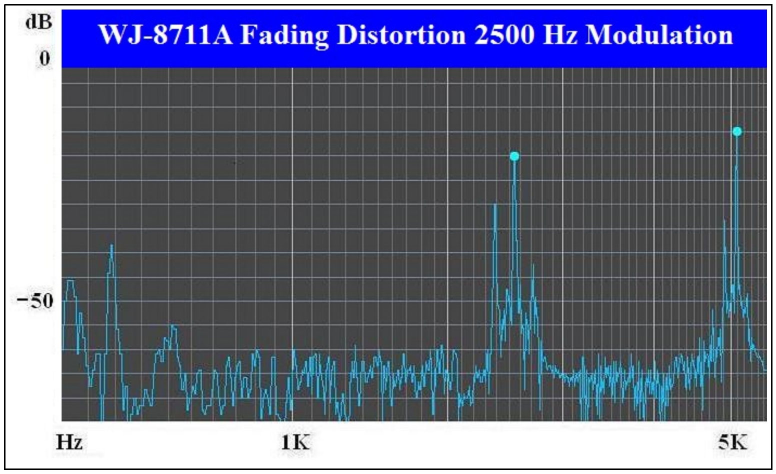
$$[(m^2A^2/8)/(kmA^2/2)]^2 * 100 = ( m^2/16k^2) * 100 = 6.25\ m^2/k^2\ .$$

For a 25 dB fade (k = 1/17.78) and 50% modulation the formula above gives 494% harmonic distortion, that is, the harmonic distortion is 6.9 dB greater than the fundamental.

Fading was simulated by sweeping an RF notch filter slowly through the 1403 kHz carrier. The receivers used for the simulation were (1) a modified R-390A in AM mode with a 6.5 kHz bandwidth tuned to 1403 kHz, and (2) a WJ-8711A in AM mode with a 6 kHz bandwidth. AGC release times for the receivers was FAST. The graphics below are instantaneous audio spectrums at the moment of a maximum carrier fade of about -25 dB. As can be seen for the R-390A (in the figure at right) the 2nd harmonic at 5100 Hz is about 5 dB greater than the fundamental, which means the harmonic distortion is about 316% . This is in reasonably good agreement with the formula above.




Note that the fading distortion for the WJ-8711A (in the figure at right) is about the same as for the R-390A (above). Also note the spurious DSP distortion 10 dB below the fundamental and 18 dB below the 2nd harmonic. Such distortion has not been observed in analog receivers.




So much for claims that DSP receivers in AM mode have vastly superior recovered AM audio in the presence of fading distortion compared to analog receivers. I have yet to use or measure a DSP receiver for which this is the case. So that there is no misunderstanding here, these results do not refer to AM synchronous detection, either analog or digital.

For a receiver with good IF filters AM fading distortion due to a diode detector can be often reduced by tuning the signal so that either the upper or lower sideband is significantly

attenuated. According to the theory above, when an AM signal is tuned so that one sideband is completely attenuated, there will be no fading distortion because audio(t) = $mA^2/4 \cos(\omega t)$.
In practice that was never the case. However, offset tuning of AM signals usually reduced harmonic distortion by substantial amounts compared to no offset tuning. For example, for an offset tuned R-390A using a 6.5 kHz BW, the simulated harmonic distortion for 2550 Hz modulation was 15 dB or more (usually more) below the fundamental for all fading levels between 0 and -25 dB fading, which is 3.2% or less (usually less) harmonic distortion. For other receivers offset tuning may reduce AM fading distortion more or less than the R-390A. For example, for an IC-746Pro with a 6500 Hz BW DSP filter and a 1000 Hz modulated signal, the harmonic distortion due to fading was in the worst case only 4 dB down , or 39%. But on the other hand, for an IC-746Pro with a 3500 Hz BW DSP filter and a 1000 Hz modulated signal, the harmonic distortion due to fading was no worse than 1.58%. Spectrum analysis of numerous fading simulations revealed that the amount of harmonic distortion varied significantly with the amount of offset tuning, the receiver IF filter BW, and their modulation frequency. These results indicate that the diode square law model together with the square wave switching model are not accurate predictors of AM fading distortion in the case of offset tuning.
In view of the above results, when using the offset tuning method, one should vary the offset and BW to minimize fading distortion and, perhaps, readjust the receiver offset and BW when program content changes.

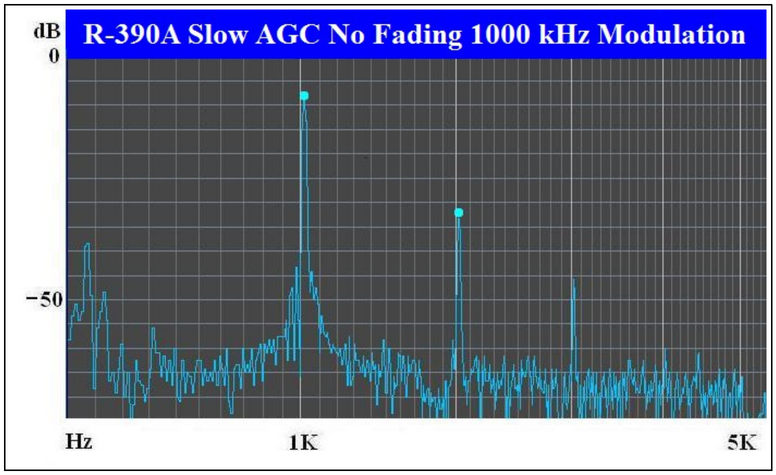
Another method of reducing AM fading distortion, which is also not widely known, is to use a sharp audio filter to attenuate the higher frequencies where much of the fading distortion is contained. Elliptic low pass audio filters (ELPAF) were specifically designed for this purpose. The ELPAF approach is currently my favorite because (a) it is relatively new, and (b) I don't have to pay attention to whether I am properly tuned to one sideband or the other. It is also useful for reducing adjacent channel splatter.

A third method of reducing AM fading distortion, well-known to most DXers, is to use ECSS (exalted carried single sideband), that is, to use SSB mode, which usually includes a product detector, with the receiver BFO zero beat (insofar as possible) to the AM carrier. If the receiver has IF filters with excellent shape factors (the IF filters significantly attenuate the AM carrier and greatly attenuate one of the sidebands), if the receiver is stable, and if the receiver can be tuned very close to zero beat, then very good recovered AM audio can be obtained. In this case, as in the case above, one of the sidebands has been attenuated by the IF filter so that there is little or no fading distortion, but here the AM carrier has been attenuated by the IF filter and replaced by the receiver BFO. A potential disadvantage of ECSS is that the recovered audio frequencies are all shifted by ABS(fc - fb) where ABS is the absolute value, fc is the AM carrier frequency (in the receiver IF passband), and fb is the BFO frequency. When the shift is only a few Hertz, it does not significantly degrade intelligibility. But in DXing situations it may not be possible to tune the BFO to within a few Hertz of the carrier. A frequent disadvantage of ECSS is that SSB filters are often quite narrow and not really suitable for recovering audio frequencies other than voice. Also, SSB filters, which tend to place the AM carrier about 20 dB down on their skirts, usually produce tinny audio due to the lower audio frequencies being cut off. Other writers frequently claim this improves intelligibility of recovered audio, but I find that not to be the case. Whether pleasure listening or DXing, I want the recovered audio to be as faithful as possible. For ECSS it is important for the BFO to be significantly greater than the AM carrier at the detector. Otherwise significant phase cancellation between the BFO and carrier can occur, causing as much or more fading distortion than in the AM mode. For for example, for a receiver with a manually tuned BFO, like the R-390A, appropriate adjustment of the BFO frequency is essential for lowest fading distortion.

A fourth method of reducing AM fading distortion, also well-known to DXers, is to use AM synchronous detection. In this case the BFO is phase locked to the AM carrier, so that even when the AM carrier is greatly reduced by fading, the BFO remains in place of the AM carrier. Since the AM carrier plus the BFO are dominant, demodulation is always via one, or the other, or both (combined), and so there is no fading distortion. ***At least that is what I believed when I wrote it. As it has turned out, more than a few AM synchronous detectors, some of them highly acclaimed, produce about as much fading distortion as ordinary diode detectors. This curious situation, which will come as a surprise to many, will be discussed in my article currently in preparation, "My Experiences With Some AM Synchronous Detectors."*** However, many AM synchronous detectors lose lock during AM carrier fades or when signals are weak, resulting in low frequency growls (hets) due to the carrier and BFO being out of lock, which can be unfortunate if such events occur during station ID's or other inopportune moments.

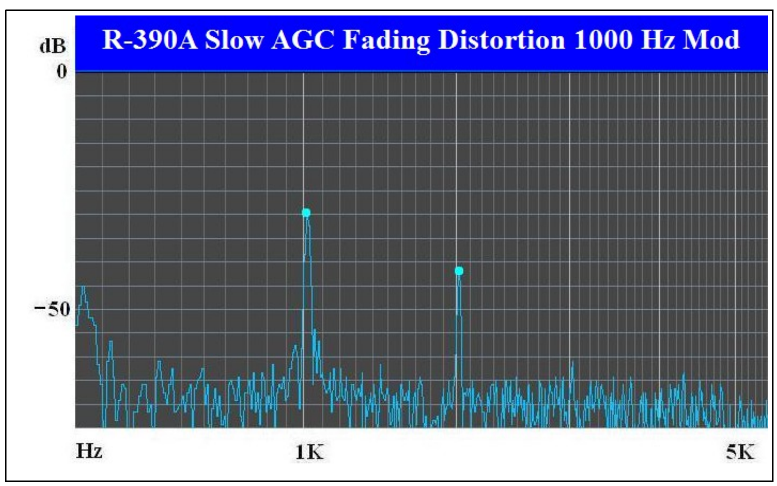
With the second, third, and fourth methods above I have observed that some residual AM fading distortion sometimes remains. There are two potential reasons for this. First, in the case of double sideband AM synchronous detection, when the AM carrier fades, the ratio of the sidebands amplitudes to the synchronized BFO amplitude changes, allowing small amounts of fading distortion to become noticeable. Consequently, improved AM fading distortion performance of AM synchronous detectors may be obtained by using audio low pass filtering. This appears to have been done for some AM synchronous detectors, such as the one in the Drake R8B. Second, when the AM carrier fades, the receiver gain distribution changes, resulting is a lower signal to noise ratio, especially for weaker signals. The result is audio which is momentarily noisier. Using a suitably slow AGC release time often significantly reduces this fading noise. Strictly speaking, fading noise is not AM fading distortion, but it does sound very much like AM fading distortion.

A fifth method, and this can hardly be overemphasized, is to use a suitably slow AGC release time, namely 2 seconds or longer. The two simulations in the first two figures below (the third simulation below is for fast AGC) are instantaneous spectrums, the first before fading, and the second at the instant of maximum fading, about -25 dB carrier fade. Before fading the 2nd harmonic was about 25 dB below the fundamental. The maximum 2nd harmonic fading distortion was about 12 dB below the fundamental, about 6.3% harmonic distortion. This is about two orders of magnitude less than the 316% harmonic distortion in the previous fading distortion simulation which used FAST AGC release and "slow motion" fading. I do not know of a good explanation for this dramatic reduction in fading distortion, but I have observed it many times in actual listening situations. If you haven't tried it, you should. You'll like it.

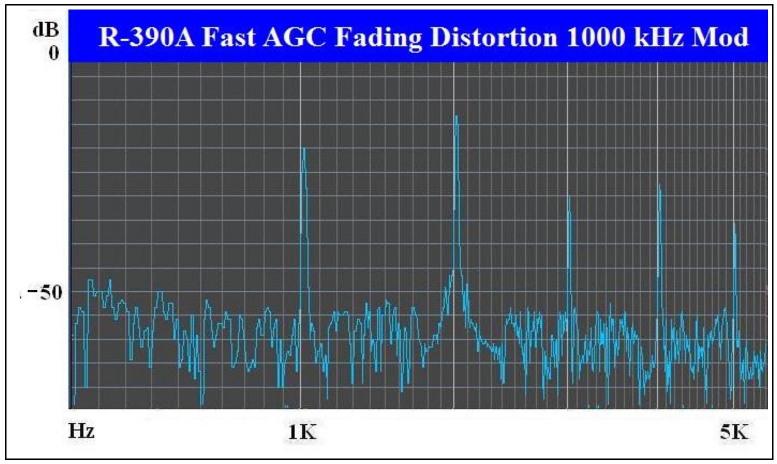



As I said above, the third fading distortion simulation below is for fast AGC. It gives you a dramatic comparison with the slow AGC fading distortion simulation at right.

Based on the results of this article, it appears that DSP receivers, receivers with AM synchronous detection, and some external AM synchronous detectors have been overrated by some, and that analog receivers, if properly used (offset tuning in AM mode or ECSS and a suitably slow AGC release time), and if equipped with sufficiently good IF filters, can provide excellent low distortion audio from fading AM signals which is only ever so slightly worse, if at all, than the corresponding audio from DSP receivers or AM synchronous detectors. With any of these methods, together with a good external audio filter like ELPAF to clean up some of the remaining distortion, a fine receiver, digital or analog, is difficult to beat.



It should be noted that the square law mathematical model of the diode detector used in this article does not give a complete model of fading distortion of AM signals as can be seen from the instantaneous audio spectrum above of a simulated fade for an R-390A below. While the square law models 2nd harmonic distortion reasonably well, it does not predict 3rd or higher order harmonic distortion due to fading of AM signals.




To model 3rd order and higher distortion terms, the detector model must include cubic and higher power terms. When a cubic power term is included, it can be shown that 3rd order distortion terms are present, and that in theory off tuning can reduce the 3rd order distortion terms by more than 30 dB. Presumably the same would be true for 4th and higher order distortion terms.